

Why Political Violence Causes Social Instability

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If protesters can coordinate, the probability that an anti-government protest turns into a successful revolution is higher under repressive than under democratic regimes. This is true for arbitrary social networks with heterogeneous agents. The implications of the provided model are illustrated using data on protests, revolutions, and political terror worldwide between 1976 and 2014.

Introduction

The spread of political activism in a society can be modeled as a diffusion process in a network (1–3), using models from epidemiology (4), physics (5), or computer science (6), most of which focus on the link between network properties and diffusion dynamics (7, 8). A crucial difference between these disciplines and the social sciences is, however, that in the latter an individual, i.e. a vertex in the underlying network, *makes a decision* on whether or not to become active based on her information and preferences (9–11), whereas in the former they simply become active if a threshold in their neighborhood is crossed. So, results in the social sciences that are derived from these models should be treated with caution: factoring in human behavior requires heterogeneous agents with potentially incomplete information who can communicate

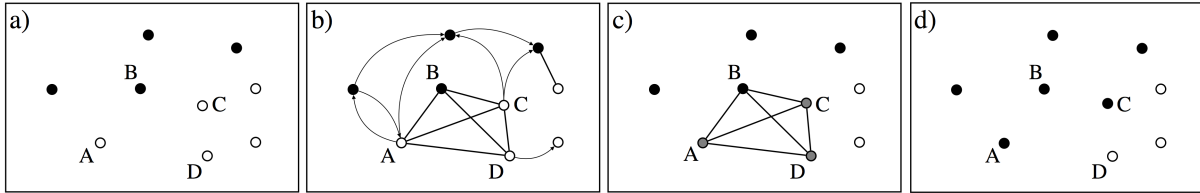


Figure 1: Suppose that initially the black vertices in a) are active, and each one is happy to be active alone; the white vertices are inactive, and each one wants to be active if they know that there are at least four other active vertices; and nobody knows that anyone else is active. In b) the straight lines depict meetings of people who can communicate and coordinate, whereas arrows depict (additional) unilateral observations without the possibility of communication. Then A, B, and C have an incentive to become active together: A knows that, if they become active, there are at least four active individuals besides her, namely B, C, and the two outside the clique she has observed; the same holds true for C. D, on the other hand, only knows about A, B, and C, so he will not become active. An “I go if you go”-commitment between A, B, and C would allow them to jointly become active, so that the new state in d) is reached. If coordination was impossible, the number of active individuals would not change.

and observe each other. In particular, agents who are close – friends, families, or partners – may coordinate and take their decision together. Figure 1 illustrates this idea.

The aim of this article is to analyse how political violence (12) and social media (13) affect the dynamic and outcome of a protest that may turn into a revolution. While restricting the access to social media slows down a protest, repression has an ambiguous effect. If a successful revolution requires some powerful individuals to become active then the probability of a revolution (given a protest) is higher under more repressive regimes. This finding, relying on the human nature to communicate and coordinate, is illustrated in Figure 2: repression increases the number of revolutions per protest.

1 Transition Probabilities

Individuals of a (finite) society N have to decide between being *active* (that is to participate in a protest) or being *inactive*. The decision is based on the anticipated outcome of the protest, which in turn depends on the (private) belief on who else is active. The belief of individual i is

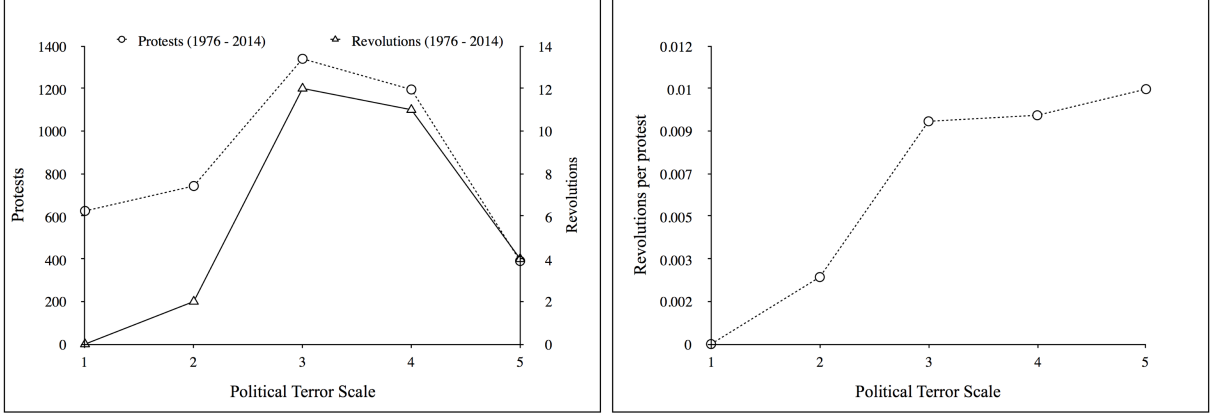


Figure 2: left: the distribution of protests and revolutions between 1976 and 2014 over the five levels of the Political Terror Scale (The PTS has five levels, level 1 countries being *under a secure rule of law* and level 5 countries with *terror expanded to the whole population*). – right: the probability that an anti-government protest turns into a revolution is 0 in PTS-1-countries and gradually increases to its maximum of about 1% in PTS-5-countries. (Data are explained in the Supplementary Material, Appendix A.)

captured by a collection \mathcal{S}_i of subsets of $N \setminus \{i\}$. A *state* consists of a set S of active individuals and a vector $\mathcal{S} = (\mathcal{S}_i)_{i \in N}$ of people’s beliefs. An individual’s expected future utility depends on her expectation about who will become active, how likely it is that the revolution will be successful, or what the consequences of a successful or failed attempt of a revolution are. All these considerations shall be contained in the (ordinal) utility function u_i , where $u_i(S')$ shall be interpreted as i ’s expected future utility if exactly the members of S' were active. Given the context it fair to assume that u_i is monotonic, i.e. the more individuals are active, the more attractive it is to become active as well. As i has only incomplete information, she must derive her utility from her belief \mathcal{S}_i . Individuals are assumed to be very risk averse, so given belief \mathcal{S}_i individual i becomes active if $u_i(S_i \cup \{i\}) \geq u_i(S_i)$ for all $S_i \in \mathcal{S}_i$.

Over time people randomly observe or meet others: observations are unilateral, whereas meetings (either physical or in social media) enable people to communicate (recall Figure 1b). Given the interpretation of meetings, it shall be assumed that if any two people meet the same

person, they meet each other as well. So, observations are modeled as directed graphs G (with vertex set N), whereas meetings are undirected subgraphs H of G (also with vertex set N) in which each connected component is a clique. Since the two are not independent of each other, I shall refer to the pair (G, H) as an *observation*.

At each point in time such an observation is randomly drawn and each individual i updates her belief: if she observes an individual j being active, she now believes that j is active; if she observes an individual k to be inactive, she now believes that k is inactive. Individuals might have (partial) information about each other's utilities, so that they can infer the behavior of (some) unobserved individuals from their observation. For instance if one observes a man being active and knows that he would not be active without his wife, one must conclude that his wife is active as well. However, people are not required to have full information about the others' utility functions, so that in general they are not certain about who is active.

Besides the ability to extract additional information from observations, there is another mechanism that crucially affects the dynamics of social unrest, namely coordination. Suppose there is a couple, both inactive and knowing that the other is inactive as well, such that given their beliefs both prefer to be inactive, but they would prefer to be active if their partner were active as well. If coordination is impossible, they will not become active – which is highly unrealistic as they probably talk and make a joint decision à la “I go if you go”. More general, if C is a clique in a meeting graph H , the members of C play a coordination game in which they are allowed to communicate. Since they have monotonic utility functions, this game has a unique strong Nash equilibrium (14) with a maximal set of active individuals, say K . I shall assume that they play this Nash equilibrium and that each $i \in C$ observes that each $j \in K$ is active (recall Figure 1c). These sets K shall be called *coordination units*. Every time a new observation is drawn, a new game is being played and the set of active individuals might change. I shall write $(S, \mathcal{S}) \xrightarrow{u}_{G,H} (T, \mathcal{T})$ if observation (G, H) causes a transition from state (S, \mathcal{S})

to state (T, \mathcal{T}) given the utility functions $u = (u_i)_{i \in N}$.

Since observations are random events, these transitions are random as well; and the probability of a transition from (S, \mathcal{S}) to (T, \mathcal{T}) is simply the probability that an observation (G, H) occurs with $(S, \mathcal{S}) \xrightarrow{u_{G,H}} (T, \mathcal{T})$. As these transition probabilities are static, they define a Markov process which starts in the state where nobody is active and everybody believes that nobody is active. The monotonicity of the utility functions imply that the process is monotonic: over time more individuals become active. (In particular, the model allows active individuals could become inactive again, but they prefer not to.) If observations are drawn according to probability distributions β or γ such that under β it is more likely that “large” groups of people observe each other or meet, then the process resulting from β will be faster than the one resulting from γ . This emphasizes the role of social media which allows exactly this kind of coordination.

The steady states of the Markov process that can be reached with positive probability are those states in which no possible observation can cause any group of individuals to change their decisions or beliefs. So, these steady states are stronger than Nash equilibria, but weaker than strong Nash equilibria: it might be profitable for some coalition to defect, but such a coalition will not meet with positive probability.

2 Political Repressions and Their Implications

The set of coalitions that are sufficiently large to overthrow the government shall be denoted by \mathcal{W} ; clearly if $S \in \mathcal{W}$ then $S' \in \mathcal{W}$ for each S' with $S \subseteq S'$. Under repression i 's utility function will be v_i with $v_i(S_i) \leq u_i(S_i)$ for all $S_i \subseteq N$ with $i \in S_i$ and $v_i(S'_i) = u_i(S'_i)$ for $S'_i \subseteq N \setminus \{i\}$. So, active individuals receive lower utility, whereas the utility of inactive individuals is not affected. However, the government cannot affect the utility function after the critical mass has been reached. Hence, $v_i(S''_i) = u_i(S''_i)$ for all $S''_i \in \mathcal{W}$.

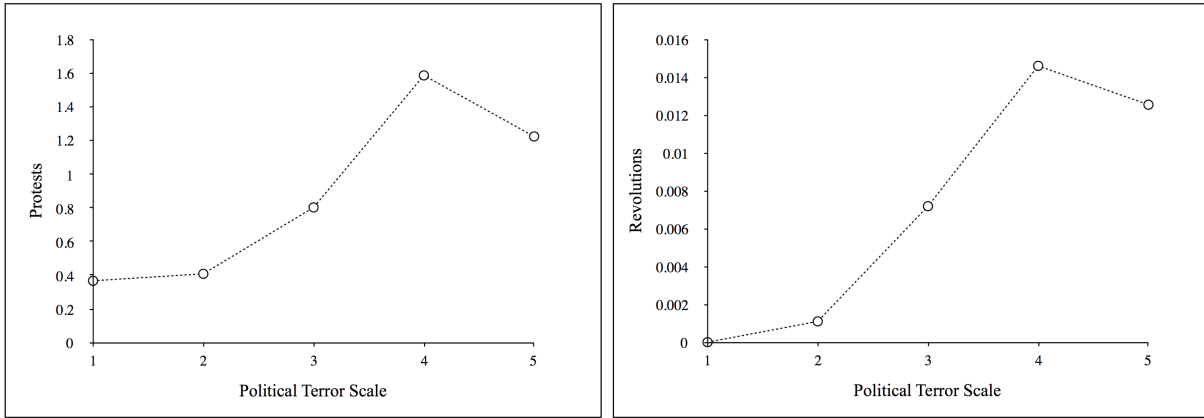


Figure 3: The numbers of protests (left) and revolutions (right) per country-year observation increase with political violence until a PTS value of 4.

Although this form of repression seems to affect activists only, its effect on protest dynamics is ambiguous. For any fixed belief an individual's decision to become active may be reversed; meanwhile observing one protester under repression can cause the belief that many more (unobserved) people are active since this protester is active despite the repression. Hence there is an indirect *informational* effect that opposes the direct effect on the utility functions. This informational effect, however, can be large only if there are already many active individuals; and it does not exist at all if there are no active individuals. So, we can expect a clear detrimental effect of political violence only if repression is sufficiently severe. This is in line with the data in Figure 3 where the number of protests increases with political repression up until PTS-level 4, and then decreases again.

Most revolutions succeeded when some very powerful players became active: the Iranian Revolution ended with the Supreme Military Council's declaration to be neutral on February 11, 1979; the Tunisian Revolution ended with the ouster of President Ben Ali by the military on January 14, 2011 (15); and the Ukrainian Revolution succeeded when the Parliament ousted President Yanukovich on February 22, 2014. These players can be introduced in my model as well: an individual i is an *opportunist* if i participates in a protest only if she believes that it will

immediately reach a critical mass, and if i 's utility function is publicly known. That is i becomes active only if for any $S_i \in \mathcal{S}_i$ it holds that $S_i \cup \{i\} \in \mathcal{W}$; and if i becomes active, everybody who observes i knows that the revolution will be successful. Note that repression cannot reverse an opportunist's decision to become active as $u_i(S_i) = v_i(S_i)$ for any all $S_i \in \mathcal{W}$. Reflecting the power of opportunists it shall be assumed that a revolution can only be successful if at least one opportunist is active.

Result In any state of the Markov process that is reached with positive probability the probability of a transition into a successful revolution is at least the same under a more repressive regime.

This result applies when a government does not change its *expected* behavior in the course of a protest. In particular, if a government is expected to violently intervene in the course of a protest (reflected by a high PTS value), the actual intervention will not have an effect on people's utility functions. An alternative way to dissolve a protest is to make concessions (16). Concessions could be interpreted as a sign of weakness causing even more people to protest (17), but here another effect is more important: opportunists may prefer to remain inactive when they are accommodated, whereas political violence will not affect their decision. Hence, loyalty of powerful individuals is important for a repressive regime to survive (18).

3 Conclusion

One would expect to see rare but large jumps in the size of anti-government protests rather than a smooth gradual increase under a repressive regime. This is in line with the observations of (12), namely that revolutions under repressive regimes are quick and unanticipated. The model also underlines the importance of social media for the process dynamics: they are the

devices that allow for the coordination of large groups and, together with the opportunists, are key to explain the instability that is caused by political violence.

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Table 1: Revolutions and Mass Protests 1976 - 2014

PTS^*	1	2	3	4	5
Revolutions	0	2	12	11	4
Protests	625	742	1339	1195	390
Country-Year Observations	1707	1821	1671	754	319
Protests per Observation	.366	.407	.801	1.585	1.223
Revolutions per Protest	0	.0027	.0090	.0092	.0103

Supplementary Material

A Data Selection

Table 1 contains the data that are depicted in Figures 2 and 3. The time frame from 1976 to 2014 is chosen, since PTS data from (19) are available since 1976 and data on protests from (20) are available until 2014.¹ Note that not for all countries these data exist for all years; each country-year combination for which both data are available is counted in Table 1. During this period (21) lists 39 events that are categorized as *Resignation of Executive due to Poor Performance or Loss of Authority*. In seven cases the political change was due to military intervention or rebel movements rather than protests,² in two cases the political leader was impeached,³ and in one case the government stepped back after a lost war.⁴ The remaining 29 events are contained in Table 1. Since these events can have a huge impact on the PTS in the year of occurrence,⁵ the latter is calculated as $PTS^* = \frac{m}{12}PTS_0 + \frac{12-m}{12}PTS_{-1}$ rounded to the closest integer. Hereby, PTS_0 is the PTS value in the year of the event, PTS_{-1} is the value in the previous year, and m is the month of the event.⁶

¹ (20) distinguishes between *Germany* and *East Germany*, while (19) distinguishes between *Germany* (from 1990), *East Germany*, and *West Germany* (until 1989). PTS data from *West Germany* in the latter data set are used for protests in *Germany* in the former.

²Bolivia 1982, East Timor 2006, Georgia 1992, Guinea-Bissau 1999, Honduras 2009, Lesotho 1990, Liberia 2003

³Lithuania 2004, Madagascar 1996

⁴Argentina 1982

⁵For instance, after the Tunisian revolution the PTS value fell from 3 in 2010 to 1 in 2011.

⁶Except for the Iranian Revolution 1979 where PTS_{-1} is not available. Here, $PTS^* = PTS_0$.

B Mathematical Appendix

The utility function u_i is *monotonic* if $u_i(S_i \cup \{i\}) \leq u_i(T_i \cup \{i\})$ for any two coalitions $S_i \subseteq T_i \subseteq N \setminus \{i\}$, and $u_i(T_i \cup K) \geq u_i(T_i)$ for all $K \subseteq N$ with $i \in K$ and all $T_i \subseteq N \setminus \{i\}$ whenever there is $S_i \subseteq T_i$ with $u_i(S_i \cup K) \geq u_i(S_i)$. Hence, if i is active her utility rises with the number of active individuals; and if a coalition $S_i \cup K$ is large enough for i to be active, so is each supercoalition T_i . A coalition S is *Nash-stable* if $u_i(S) \geq u_i(S \setminus \{i\})$ for all $i \in S$. A coalition that is not Nash-stable cannot be sustained as at least one individual has a reason to leave it immediately.

An individual i may have (partial) information about the utilities of others (or information about the information about the utilities of others), allowing her to draw inferences from any observation. For instance, she might know that some individual j will only become active if another individual k is active as well. Denote by $\mathcal{S}_i^* \subseteq 2^{N \setminus \{i\}}$ the collection of sets (without i) that i *considers* Nash stable.⁷ The only restriction shall be that \mathcal{S}^* contains all Nash-stable coalition without i , that is if S is Nash stable then i cannot assume it is not. It should be noted that \mathcal{S}_i^* depends on the players' utility functions, that is $\mathcal{S}_i^* = \mathcal{S}_{i,u}^*$.

A *belief* of player i is a collection of sets $\mathcal{S}_i \subseteq \mathcal{S}_i^*$. The two implicit assumptions here are that i always knows whether she is active, and that (she thinks) she can change her status without being observed by others. As individuals do not exactly know who is active, their decisions must be based on their beliefs. I shall assume that individuals are pessimistic with respect to the number of active individuals, so they become active if and only if $u_i(S_i \cup \{i\}) \geq u_i(S_i)$ for all $S_i \in \mathcal{S}_i$.

Individuals constantly observe their environment and update their beliefs. Suppose that individual i thinks that S_i is the true state of the world, and that she observes the members of

⁷If i has no information at all then $\mathcal{S}_i^* = 2^{N \setminus \{i\}}$.

$C \subseteq N \setminus \{i\}$ becoming active, and those of $D \subseteq N \setminus (C \cup \{i\})$ inactive. Then i must now believe that the true state of the world lies in the set

$$\Phi_{S_i}(C, D) = \{T_i \in \mathcal{S}_i^* : (S_i \setminus D) \cup C \subseteq T_i \subseteq N \setminus D\}.$$

Suppose that individual i with belief \mathcal{S}_i discovers that the members of $A \subseteq N \setminus \{i\}$ have been active, those of $B \subseteq N \setminus (A \cup \{i\})$ inactive. She then verifies which of her beliefs might have been true, namely of those $S_i \in \mathcal{S}_i$ with $A \subseteq S_i \subseteq N \setminus B$. The overall belief updating process can, hence, be captured by the function

$$\Phi_{\mathcal{S}_i}^u(A, B, C, D) = \bigcup_{S_i \in \mathcal{S}_i: A \subseteq S_i \subseteq N \setminus B} \Phi_{S_i}(C, D).$$

Note that, in general, the updated belief might be an empty set; in this paper, however, the non-emptiness is guaranteed (see the Lemma below).

Let (S, \mathcal{S}) be a state and let (G, H) be an observation. Denote by G_i, H_i be the neighbors of i in the graphs G and H , i.e. G_i the set of individuals that i observes, and $H_i \subseteq G_i$ is the set of individuals that i meets. The players in H_i (who all meet) play a simultaneous move game with strategies being active or inactive. In order to find the payoffs in this game define for $K \subseteq H_i$ and $j \in H_i$

$$\Psi_j(K) = \Psi_{S, S_j, u}^{G, H}(K) = \Phi_{\mathcal{S}_j}(G_j \cap S, G_j \setminus S, K, \emptyset)$$

That is, $R \in \Psi_j(K)$ if R is consistent with i 's observation that the members of $G_i \cap S$ have been active, those of $G_i \setminus S$ have been inactive, the members of K are becoming active, and nobody is becoming inactive.⁸ It is easy to see that $\Psi(K) \subseteq \Psi(K')$ whenever $K' \subseteq K$. Hence, if $u_i(T'_i \cup \{i\}) \geq u_i(T'_i \setminus R')$ for all $R' \subseteq H_i$ and all $T'_i \in \Psi(K')$ then $u_i(T_i \cup \{i\}) \geq u_i(T_i \setminus R)$

⁸This belief is an *ex ante* belief: once the equilibrium is being played, further information might be revealed, namely that the members of $(H_i \cap S) \setminus K$ have become inactive. Whether or not individuals take that into account after the game has been played is not relevant for the results of this paper, as it will turn out that no active individual will become inactive again (see the Lemma below).

for all $R \subseteq H_i$ and all $T_i \in \Psi(K)$ by the monotonicity of u_i . This implies that there is a unique largest strong Nash equilibrium K_i in the game that is played among H_i . The transition $(S, \mathcal{S}) \rightarrow_{G,H} (T, \mathcal{T})$ is, therefore, well-defined with $T = \bigcup_{i \in N} K_i$ and $\mathcal{T}_i = \Psi_i(K_i)$ for all $i \in N$. For two states (S, \mathbf{S}) and (T, \mathbf{T}) the probability of a transition from the former to the latter (given a vector of utility functions u) is given by

$$\mu_{S,\mathbf{S}}^{T,\mathbf{T}} = \sum_{G,H:(S,\mathbf{S}) \xrightarrow{u}_{R,Q} (T,\mathbf{T})} \gamma(G,H), \quad (1)$$

where $\gamma(G,H)$ is the probability that observation (G,H) occurs. These transition probabilities define a Markov process over the set of states. One can define an order \supseteq on that set with $(S, \mathcal{S}) \supseteq (T, \mathcal{T})$ if $T \subseteq S$. The following lemma proves the Markov process is monotonic with respect to that order if it starts in the state where nobody is active and everybody believes that nobody is active (this state satisfies the premise of the lemma).

Lemma Let (S, \mathcal{S}) be a state with $S \in \mathcal{S}_i \subseteq \mathcal{S}_i^*$ for all $i \in N$. Suppose that for all $\emptyset \neq R \subseteq N$ there is $j \in R \cap S$ with $u_j(S_j \cup \{j\}) \geq u_j(S_j \setminus R)$ for all $S_j \in \mathcal{S}_j$. Then for each observation (G,H) and any state (T, \mathcal{T}) with $(S, \mathcal{S}) \xrightarrow{u}_{G,H} (T, \mathcal{T})$ it holds that $S \subseteq T$ and $T \in \mathcal{T}_i \subseteq \mathcal{S}_i^*$ for all $i \in N$. Moreover, for each $\emptyset \neq R \subseteq N$ there is $i \in R \cap T$ with $u_i(T_i) \geq u_i(T_i \setminus R)$ for all $T_i \in \mathcal{T}_i$.

Proof. Let (S, \mathcal{S}) have the required properties, let (G,H) be an observation and let (T, \mathcal{T}) satisfy $(S, \mathcal{S}) \xrightarrow{u}_{G,H} (T, \mathcal{T})$. Let $i \in N$ and let K_i be the set of players who choose being active in the largest strong equilibrium of the game played within H_i . By construction $\mathcal{T}_i \subseteq \mathcal{S}_i^*$, and for each $T_i \in \mathcal{T}_i$ there is $S_i \in \mathcal{S}_i$ with $S_i \subseteq T_i$. If $i \in S$ then for each $S_i \in \mathcal{S}_i$ it holds that $u_i(S_i \cup \{i\}) \geq u_i(S_i)$. Hence, for each $T_i \in \mathcal{T}_i$ it holds that $u_i(T_i \cup \{i\}) \geq u_i(T_i)$ by the monotonicity of u_i . Therefore, again by the monotonicity of u_i , $i \in K_i \subseteq T$, so that $S \subseteq T$. Since $G_i \cap S \subseteq S \subseteq N \setminus (G_i \setminus S)$ and $T \in \mathcal{S}_i^*$ and $S \cup K \subseteq T$

it holds that $T \in \mathcal{T}_i$. Let $R \subseteq N$ and assume that for all $j \in R \cap T$ there is $T_j \in \mathcal{T}_j$ with $u_j((T_j \setminus R) \cup ((T_j \cup \{j\}) \cap R)) = u_j(T_j \cup \{j\}) < u_j(T_j \setminus R)$. Then the monotonicity of u_j implies $u_j(S_j \cup \{j\}) \leq u_j((S_j \setminus R) \cup ((T_j \cup \{j\}) \cap R)) < u_j(S_j \setminus R)$ for all $S_j \in \mathcal{S}_j$ with $S_j \setminus R \subseteq T_j \setminus R$. Since such S_j exists by construction this is a contradiction to the premise of the Lemma. *Q.E.D.*

One can define a partial order \succeq on the set of observations by setting $(G^1, H^1) \succeq (G^2, H^2)$ for two observations (G^1, H^1) and (G^2, H^2) with G_2 being a subgraph of G_1 and H_2 being a subgraph of H_1 . Let β and γ are distributions over the set of observations such that β first order stochastically dominates γ with respect to the order \succeq , and let μ and ν be the corresponding transition probabilities. Then $\mu_{S, \mathcal{S}}^{\dots}$ first order stochastically dominates $\nu_{S, \mathcal{S}}^{\dots}$ with respect to \triangleright for any (S, \mathcal{S}) that is reached with positive probability. Let the corresponding processes be denoted by X and Y . Since they are monotonic, a result from (22) implies that $X(t)$ first order stochastically dominates $Y(t)$ for all $t \geq 0$.

I shall now turn to the proof of the main result.

Proof of the Result Let u and v be two vectors of utility functions such that u is the status quo and v is the result of more repressions, and let the transition probabilities (as in Equation (1)) be given by μ and λ , respectively. Since the process starts at a state (\emptyset, \mathcal{S}) with $\emptyset \in \mathcal{S}_i$ for all $i \in N$, all states that are reached with positive probability satisfy the premise of the Lemma. Let u and v be as described. It must be proven that

$$\sum_{(T, \mathcal{T}): T \in \mathcal{W}} \mu_{S, \mathcal{S}}^{T, \mathcal{T}} \leq \sum_{(T, \mathcal{T}): T \in \mathcal{W}} \lambda_{S, \mathcal{S}}^{T, \mathcal{T}}$$

for any state (S, \mathcal{S}) . For this purpose let (T, \mathcal{T}) be a state with $T \in \mathcal{W}$, and let (G, H) be such that $(S, \mathcal{S}) \rightarrow_{G, H}^u (T, \mathcal{T})$. If such (G, H) does not exist then $\mu_{S, \mathcal{S}}^{T, \mathcal{T}} = 0 = \lambda_{S, \mathcal{S}}^{T, \mathcal{T}}$. Otherwise, $T = \bigcup_{i \in N} K_i$, where K_i are defined as before. Let $i \in T$ be an opportunist. Since the identity

and utility function of i are publicly known, it must hold that $\mathcal{T}_j \subseteq \mathcal{W}$ for all $j \in K_i$. Hence, for any $T_j \in \mathcal{T}_j$ and any $K' \subseteq K$ there is $j \in K'$ such that

$$v_j(T_j) = u_j(T_j) \geq u_j(T_j \setminus K') = v_j(T_j \setminus K')$$

where the first and the last equation come from the definition of v_j . This means that the largest strong Nash Equilibrium in H_i with utility functions v must contain K_i as active individuals. Hence, if (T', \mathcal{T}') is such that $(S, \mathcal{S}) \xrightarrow{v}_{G,H} (T', \mathcal{T}')$ then $\mathcal{T}'_i \subseteq \mathcal{W}$ as before and $T' \in \mathcal{T}'_i$ by the Lemma. So, if (G, H) causes a transition from (S, \mathcal{S}) to a state (T, \mathcal{T}) with $T \in \mathcal{W}$ under u , it causes a transition from (S, \mathcal{S}) to a state (T', \mathcal{T}') with $T' \in \mathcal{W}$ under v . This proves the claim. *Q.E.D.*